Dimensions, conversions, formula's and tabels for designing a laminar water nozzle

FLOW, DIAMETER AND VELOCITY

For conversion purposes, I will use these notations and equations: 1 gallon = 3.78 l, 1 foot = 1'=12''=305 mm, 1'=25.4 mm. This means that the typical diameter of a household piping (13 mm) is about 0.5 inch = 0.5''

Typical flow of a household water tap in the Netherlands: 10 l/min = 0.17 l/sec = 600 l /hour = 17 $10^{-5}m^3$/s (2.6 Gallon/min = 159 Gallon/hour)

Flow of Gardena 6000 pump, 220 Watt: 6 m^3 /hour = 6000 l/hour = 100 l /min = 1.7 l/sec. So, the flow from this simple pump could be 10 times higher than the flow from the household water tap.

Typical diameter of household piping is 13 mm. Pipes of 10 or 8 mm are also used.

Most of the nozzles diameters in the forum are 3 inch, 4 inch, 6 inch and 8 inch. In the Netherlands hardware stores, diameters of PVC tubes are 75mm (3''), 110 mm (4.3''), 125 mm (5''). More specialized shops can offer other diameters like 160 mm (6'')

REYNOLDS NUMBERS

The Reynolds number (Re) can be computed for a given liquid, flow and shape of the tube to predict if the flow will be laminar or turbulent. For a cylindrical shaped tube and water the Reynolds number is computed as: Re=(diameter tube x mean velocity / kinematic viscosity). The kinematic viscosity for water is 1.0 $10^{-6}m^2$/s at room temperature. For our purposes it is easier to express the Reynolds number as a function of the flow and the diameter: Re=flow /(kinematic velocity x diameter x 3.14)

Using the flow of a household tap as a reference, 10l /min, a table can be computed that shows diameters, the area and mean velocity of the water and the Reynolds number for different diameters.

diameter	Area (mm^2)	Mean velocity (m/s) assuming a flow of 10l/min	Reynolds number
8 mm	201	0.84	6625
10 mm	314	0.54	5233
13 mm (0.5'')	531	0.33	4076
75 m (3'')	17662	9.62 E-3	697
110 mm (4.3'')	37794	4.49 E-3	481
125 mm (5'')	49062	3.46 E-3	424
160 mm (6'')	80384	2.11 E-3	331
8'' (203mm)	129396	1.31 E-3	261

12’’ (304 mm)	290186	0.585 E-3	174

The flow is laminar for RE less than 2000 and turbulent for values greater than 4000. This means that the flow from a household tap is turbulent if the tap is opened completely.

PRESSURE AND POWER

Typical pressure on a household water tap: between 2 and 8 bar

Atmospheric pressure is 1 bar and is the pressure felt under water at a depth of 10 m. The pressure from a water volume of 1 m height is .1 bar = 10000 Pa. Pressure loss per m due to gravity is 0.1 bar

Gardena 6000 pumps up to 5 m. So, the pressure must be more than .5 bar

The power (P) that’s needed to pump a volume of water per unit of time (Q) through a pipe with a difference of pressure between inlet and out let (p) is given by P=Qp

The Gardena 6000 pump can, according to its specs, pump 6000l/hour from a depth of 5 m. In this case Q=1.7 $10^{-3} m^3$/s and p = .5 bar = .5 10^5Pa. P required is 85 Watt. This is the mechanical energy that is required per time unit. Due to conversion losses we need more electrical power for achieving this result.

From experimental data is found that pressure loss in piping is linear with length and $diameter^{-5}$ (Hazen-Williams formula). This means that we can get much better efficiency from out pumps by using tubes with larger diameters than the standard 0.5’’.

From these results we can conclude that we will not get a maximum flow from our Gardena pump if we use standard 0.5’’ tubing

ARCH’S

In order to find out what is needed to get a water arch with a specified height (H), width (W) we need to compute the velocity of a small water volume (v) when it leaves the nozzle. Using this velocity and the diameter of the jet we can compute the flow (Q) that is needed and estimate the diameter of the nozzle by keeping the Reynolds number sufficiently low.

Given the angle between the jet and the ground (φ), the gravity acceleration (g = 10 m/s^2) and the mean velocity of the water (v), the equations for the horizontal and vertical position (X,Y) as function of the time are given by: X(t)= v cos(φ) t , Y(t)= -0.5gt^2+ v sin(φ) t . The vertical velocity (V) is given by: V = v sin(φ) – gt = 0. At the top of the arch the vertical velocity (V) is zero. This happens when t= v sin(φ)/g. The height of the arch (H) is then given by v^2sin(φ) sin(φ) / 2g , while the vertical position of the top of the arch is given by v^2sin(φ) cos(φ) /g. The width of the arch (W) is 2v^2sin(φ) cos(φ) /g

DESIGN STRATEGY

At this moment we have no theory that describes the flow in a nozzle that has been filled up with straws. A practical approach is to treat the nozzle as a tube and to choose a value for the flow that results in a value for the Reynolds number that predicts a laminar flow, say 2000. Then, if we assume that the flow is fixed to a value that the flow is laminar in the nozzle (Q), we can find relations between the diameter of the nozzle exit and the width and height of the arch.

Q = v$\pi\ d^2/4$, -> $v = 4Q\ /\pi\ d^2$ - > H = $\frac{8Q^2 \sin\varphi \quad \sin\varphi}{g\pi^2 d^4}$ and W = $\frac{32Q^2 \sin\varphi \cos\varphi}{g\pi^2 d^4}$

If we assume an angle of 45 degrees, these formulas can be simplified to: H=$\frac{Q^2}{25\ d^4}$ and W= 4 H

Then, we can make tables for the different diameters of the nozzles that shows theoretical values for the flow and the height, assuming an angle of 45 degrees and two values for the diameters of the water jet 0.5" (13mm) , 8 mm and 0.25 " (6.5 mm).

Nozzle diameter	Number of straws (6mm)	Flow in $10^{-6} m^3/s$ Re=2000	Flow Re=2000	Height d=0.5" d=13mm	Height d=10mm	Height d=8mm	Height, d=0.25" d=6.5mm
75 mm (3")	156	471	27 l/min	0.31	0.89	2.17	4.97
110 mm (4.3")	335	690	40 l/min	0.67	1.90	4.65	10.7
125 mm (5")	433	785	46 l/min	0.86	2.46	6.02	13.8
160 mm (6")	709	1005	59 l/min	1.41	4.04	9.86	22.6
8" (203mm)	1144	1274	75 l/min	2.27	6.49	15.9	36.4
12" (304 mm)	2563	1909	112 l/min	5.10	14.6	35.6	81.6

The width of the arch is 4 time the height for an angle of 45 degrees. The number of straws with a diameter of 3 mm is 4 times higher.

Note that the values in the table are theoretical values. We know that in practice these numbers are much lower. One problem in practice might be that the pump cannot provide sufficient pressure. The power in watts, should be much more than Q H 10^4 just to bring the flow of water Q to the height H. We don't know how much the pressure drop is in the nozzle.

I hope this helps. Comments are welcome!

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